International Journal of Engineering Research & Management Technology
ISSN: 2348-4039
September- 2014 Volume 1, Issue-5
Email: editor@ijermt.org www.ijermt.org

Note on Order Level Inventory Model with Increasing Demand and Variable Deterioration

Hari Kishan
Department of Mathematics
D.N. College
Meerut (UP)

Megha Rani
Department of Mathematics
RKGIT
Ghaziabad (UP)

Sarvendra Saroha
Research scholar
D.N. College
Meerut (UP)

ABSTRACT:
In this paper, an order level inventory model for increasing demand and variable deterioration. Demand rate has been considered as linear function of time and deterioration rate of power pattern form. Cost minimization technique has been used.

Key-words: Order level inventory, increasing demand and power pattern.

1 INTRODUCTION:

Ghare & Schrader (1963), **Aggrawal** (1978) and **Dave & Patel** (1981) discussed the inventory models with constant rate of deterioration. **Covert & Philip** (1973), **Philip** (1974), **Chowdhury, Roy and Chaudhuri** (1983), **Dave** (1986) and **Bahari-Kashani** (1989) developed the inventory models with time dependent deterioration and instantaneous replenishment. **Mishra** (1975), **Mandal & Phaujdar** (1989) and **Dave & Chaudhuri** (1986) developed the inventory models with finite rate of production and time dependent deterioration.

Vandana Dixit & Shah (2006) developed the inventory model under the assumption of time dependent deterioration with production rate proportional to demand which is decreasing function of time. But their mathematical analysis is not correct. In this paper, the mathematical incorrectness of **Dixit & Shah (2006)** has been corrected with increasing demand rate in place of decreasing demand rate.

2 ASSUMPTIONS AND NOTATIONS:

The present inventory model has been developed under the following assumption and notations.

1. The demand rate d(t) =a+bt, a and b are positive constants, is an increasing function of time.
2. The replenishment rate is r(t)= γ d(t), where $\gamma \succ 1$ is a constant.
3. The deterioration rate is $\theta(t) = \alpha\beta t^{\beta-1}, 0 < \alpha < 1\ ,\ t < 0\ ,\beta \geq 1$.
4. The lead time is zero and shortages are not allowed
5. The deteriorated items are neither replaced nor repaired during the cycle time.
6. C is unit cost, C_1 is the unit holding cost per time , A is the ordering cost which is fixed, T is the cycle time and TC is the total cost per unit time.

3 MATHEMATICAL MODELS:

Let $q(t)$ be the inventory level at any time t, $0 \leq t \leq T$. The stock level is zero initially at the beginning of production time i.e. $q(0) = 0$. It continues up to the time $t = t_1$ and stock with stock level S. Finally, the stock level reduces to zero at t=T due to demand and deterioration. This completes one cycle. This model has been represented in figure 1:

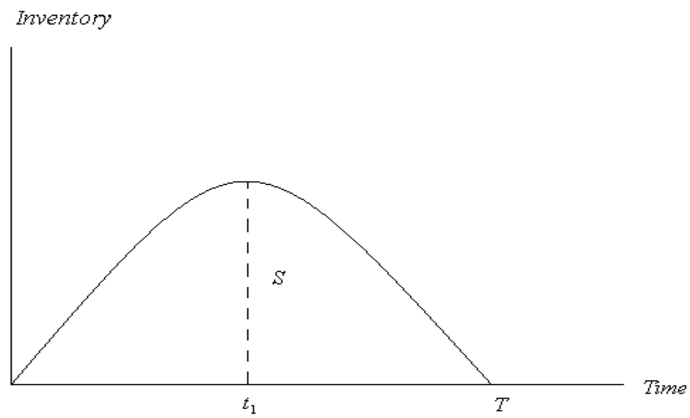

Figure 1

The governing differential equations of inventory system in the interval $[0,T]$ are given by.

$$\frac{dq}{dt}+\theta(t)q=r(t)-d(t), \qquad 0\le t\le t_1 \qquad \ldots(1)$$

and

$$\frac{dq}{dt}+\theta(t)q=-d(t), \qquad t_1\le t\le T \qquad \ldots(2)$$

The objective is to obtain the optimal values of S and TC subject to the decision variables t_1 and T. By the substitution of values of $d(t)$, $r(t)$ and $\theta(t)$ the equation (1) and (2) become

$$\frac{dq}{dt}+\alpha\beta t^{\beta-1}q=(\gamma-1)(a+bt), \qquad 0\le t\le t_1 \qquad \ldots(3)$$

and

$$\frac{dq}{dt}+\alpha\beta t^{\beta-1}q=-(a+bt), \qquad t_1\le t\le T \qquad \ldots(4)$$

The solution of equation (3) (by neglecting the higher powers of α) is given by

$$qe^{\alpha t^{\beta}}=(\gamma-1)\left(at+\frac{bt^2}{2}+\frac{a\alpha t^{\beta+1}}{\beta+1}+\frac{b\alpha t^{\beta+2}}{\beta+2}\right)+c_1.$$

Using the initial condition in the above equation, we get c_1 =0 and consequently the stock level is given by

$$q=(\gamma-1)\left[at+\frac{bt^2}{2}+\frac{a\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha b t^{\beta+2}}{\beta+2}\right]e^{-\propto t^{\beta}}, \ 0\le t\le t_1 \qquad \ldots(5)$$

Similarly the solution of equation (4) is given by.

$$qe^{\propto t^{\beta}}=-\left(at+\frac{bt^2}{2}+\frac{a\alpha t^{\beta+1}}{\beta+1}+\frac{b\alpha t^{\beta+2}}{\beta+2}\right)+c_2.$$

The condition $q(t_1)=S$, we get.

$$Se^{\alpha t_1^{\beta}}=-\left(at_1+\frac{bt_1^{\,2}}{2}+\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{b\alpha t_1^{\beta+2}}{\beta+2}\right)+c_2.$$

This gives

$$c_2=S\left(1+\alpha t_1^{\,\beta}\right)+\left(at_1+\frac{bt_1^{\,2}}{2}+\frac{a\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha b t_1^{\beta+2}}{\beta+2}\right).$$

Therefore

$$q(t) = -\left(at + \frac{bt^2}{2} + \frac{a\alpha t^{\beta+1}}{\beta+1} + \frac{\alpha b t^{\beta+2}}{\beta+2}\right)e^{-\alpha t^{\beta}}$$

$$+ S\left(1+\alpha t_1^{\beta}\right)e^{-\alpha t^{\beta}} + \left(at_1 + \frac{bt_1^2}{2} + \frac{a\alpha t_1^{\beta+1}}{\beta+1} + \frac{b\alpha t_1^{\beta+2}}{\beta+2}\right)e^{-\alpha t^{\beta}}. \qquad t_1 \le t \le T$$

…(6)

Using the condition $q(T) = o$ in (6) we get

$$S = \frac{1}{\left(1+\alpha t_1{}^{\beta}\right)}\left[a\left(T - t_1 + \frac{\alpha T^{\beta+1}}{\beta+1} - \frac{\alpha t_1{}^{\beta+1}}{\beta+1}\right)\right.$$

$$\left. + b\left(\frac{T^2 - t_1{}^2}{2} + \alpha\left(\frac{T^{\beta+2} - t_1{}^{\beta+2}}{\beta+2}\right)\right)\right]. \qquad \ldots(7)$$

Now the average holding cost is given by

$$HC = \frac{C_1}{T}\left[\int_0^{t_1} q\,dt + \int_{t_1}^{T} q\,dt\right]$$

$$= \frac{C_1}{T}\ \Big[\ \int_0^{t_1}(\gamma-1)\left[at + \frac{bt^2}{2} + \frac{a\alpha t^{\beta+1}}{(\beta+1)} + \frac{\alpha b t^{\beta+2}}{\beta+2}\right]\left[1-\alpha t^{\beta}\right]dt$$

$$+ \int_{t_1}^{T_1}[S\left(1-\alpha t_1^{\beta}\right) + a(t_1 - t) + \frac{b}{2}\left(t_1^2 - t^2\right)$$

$$+ \frac{a\alpha}{\beta+1}\left(t_1^{\beta+1} - t^{\beta+1}\right) + \frac{\alpha b}{\beta+2}\left(t_1^{\beta+2} - t^{\beta+2}\right)]\left(1-\alpha t^{\beta}\right)dt\Big]$$

$$= \frac{C_1}{T}\left[\frac{a}{2}\left\{(\gamma-1)t_1^2 - \frac{2\alpha\beta(\gamma-1)t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{(\gamma-1)\alpha^2 t_1^{2\beta+2}}{(\beta+1)^2}\right.\right.$$

$$\left. + 2t_1T - T^2 - t_1^2 + \frac{2\alpha}{\beta+1}\left(t_1^{\beta+1}T - \frac{T^{\beta+2}}{(\beta+2)} - \left(\frac{\beta+1}{\beta+2}\right)t_1^{\beta+2}\right)\right\}$$

$$+ \frac{b}{2}\left\{\frac{(\gamma-1)t_1^3}{3} - \frac{\alpha\beta(\gamma-1)t_1^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{2\alpha^2(\gamma-1)t_1^{2\beta+3}}{(\beta+2)(\beta+3)}\right.$$

$$\left. + t_1^2T - \frac{T^3}{3} - \frac{2}{3}t^3 - \frac{2\alpha}{(\beta+2)}\left(t_1^{\beta+2}T - \frac{T^{\beta+3}}{\beta+3} - \frac{(\beta+2)}{(\beta+3)}t_1^{\beta+3}\right)\right\}$$

$$\left. S\left(1-\alpha t_1^{\beta}\right)\left(T - t_1 - \frac{\alpha}{\beta+1}\left(T^{\beta+1} - t_1^{\beta+1}\right)\right)\right]. \qquad \ldots(8)$$

The average deterioration cost per unit time is given by

$$DC = \frac{C}{T}\left[\int_0^{t_1} r(t)dt - \int_0^{T} d(t)dt\right]$$

$$= \frac{C}{T}\left[\gamma\int_0^{t_1}(a+bt)dt - \int_0^{T}(a+bt)dt\right]$$

$$=\frac{C}{T}\left[a(\gamma t_1-T)+\frac{b}{2}\left(\gamma t_1^2-T^2\right)\right]. \qquad \ldots(9)$$

The average cost of the inventory system is given by

TC=HC+DC+A. …(10)

$$=\frac{C_1}{T}\left[(\gamma-1)\left[\frac{at_1^2}{2}+\frac{bt_1^3}{6}-\frac{a\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{b\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}\right.\right.$$

$$\left.-\frac{a\alpha^2t_1^{2\beta+2}}{2(\beta+1)^2}-\frac{b\alpha^2t_1^{2\beta+3}}{(\beta+2)(\beta+3)}\right]+S\left(1-\alpha t_1^{\beta}\right)(T-t_1)+a\left[t_1T-\frac{1}{2}\left(T^2+t_1^2\right)\right]$$

$$+\frac{b}{2}\left[t_1^2T-\frac{T^3}{3}-\frac{2}{3}T_1^3\right]+\frac{a\alpha}{(\beta+1)}\left(t_1^{\beta+1}T-\frac{T^{\beta+2}}{(\beta+2)}-t_1^{\beta+2}\frac{\beta+1}{\beta+2}\right)$$

$$-\frac{\alpha b}{\beta+2}\left(t_1^{\beta+2}T-\frac{T^{\beta+3}}{\beta+3}-\frac{(\beta+2)^{\beta+3}}{(\beta+3)}t_1^{\beta+3}\right)]$$

$$+\frac{C}{T}\left[a(\gamma t_1-T)+\frac{b}{2}\left(\gamma t_1^2-T^2\right)\right]$$

$$=\frac{C_1}{T}\left[\frac{a}{2}\left\{(\gamma-1)t_1^2-\frac{2\alpha\beta(\gamma-1)t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{(\gamma-1)\alpha^2t_1^{2\beta+2}}{(\beta+1)^2}\right.\right.$$

$$\left.+2t_1T-T^2-t_1^2+\frac{2\alpha}{\beta+1}\left(t_1^{\beta+1}T-\frac{T^{\beta+2}}{(\beta+2)}-\left(\frac{\beta+1}{\beta+2}\right)t_1^{\beta+2}\right)\right.$$

$$+\frac{b}{2}\left\{\frac{(\gamma-1)t_1^3}{3}-\frac{\alpha\beta(\gamma-1)t_1^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{2\alpha^2(\gamma-1)t_1^{2\beta+3}}{(\beta+2)(\beta+3)}\right.$$

$$\left.\left.+t_1^2T-\frac{T^3}{3}-\frac{2}{3}t^3-\frac{2\alpha}{(\beta+2)}\left(t_1^{\beta+2}T-\frac{T^{\beta+3}}{\beta+3}-\frac{(\beta+2)}{(\beta+3)}t_1^{\beta+3}\right)+S\left(1-\alpha t_1^{\beta}\right)(T-t_1)\right]\right.$$

$$+\frac{C}{T}\left[a(\gamma t_1-T)+\frac{b}{2}\left(\gamma t_1^2-T^2\right)\right]. \qquad \ldots(11)$$

Now for the optimal values of TC, we have to differentiate TC w.r.t. t_1 and T partially and put them equal to zero. Thus we have

$$\frac{C_1}{T}\left[a\left\{(\gamma-1)t_1-\frac{\alpha\beta(\gamma-1)t_1^{\beta+1}}{(\beta+1)}-\frac{(\gamma-1)\alpha^2t_1^{2\beta+1}}{(\beta+1)}+T-t_1+\frac{\alpha}{\beta+1}\left(t_1^{\beta}T-t_1^{\beta+1}\right)\right\}\right.$$

$$+\frac{b}{2}\left\{(\gamma-1)t_1^2-\frac{\alpha\beta(\gamma-1)t_1^{\beta+2}}{(\beta+2)}-\frac{2\alpha^2(2\beta+3)(\gamma-1)t_1^{2\beta+2}}{(\beta+2)(\beta+3)}\right.$$

$$\left.\left.+2t_1T-2t_1^2-2\alpha\left(t_1^{\beta+1}T-t_1^{\beta+2}\right)\right\}+S\left(\alpha(\beta+1)t_1^{\beta}-\alpha\beta t_1^{\beta-1}T-1\right)\right].$$

$$+\frac{C(a+b)\gamma}{T}=0. \qquad \ldots(12)$$

$$-\frac{C_1}{T^2}\left[\frac{a}{2}\left\{(\gamma-1)t_1^2-\frac{2\alpha\beta(\gamma-1)}{(\beta+1)(\beta+2)}-\frac{(\gamma-1)\alpha^2t_1^{2\beta+2}}{(\beta+1)^2}\right.\right.$$

$$+T^2-t_1^2+\frac{2\alpha}{\beta+1}\left(-\frac{\beta+2}{(\beta+2)}-\left(\frac{\beta+1}{\beta+2}\right)t_1^{\beta+2}\right)$$
$$+\frac{b}{2}\left\{\frac{(\gamma-1)t_1^3}{3}-\frac{\alpha\beta(\gamma-1)t_1^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{2\alpha^2(\gamma-1)t_1^{2\beta+3}}{(\beta+2)(\beta+3)}\right.$$
$$\left.+\frac{2T^3}{3}-\frac{2}{3}t_1^3-\frac{2\alpha}{(\beta+2)}\left(T^{\beta+3}-\frac{(\beta+2)}{(\beta+3)}t_1^{\beta+3}\right)\right\}+S\left(1-\alpha t_1^{\beta}\right)t_1\Big].$$
$$-\frac{C}{T^2}\left[a\gamma t_1+\frac{b}{2}\gamma t_1^2\right]-\frac{bC}{2}=0. \qquad \ldots(13)$$

The values of t_1 and T obtained from (12) and (13) will be optimal provided they satisfy the inequalities

$$\frac{\partial^2 TC}{\partial t_1^2}>0, \qquad \frac{\partial^2 TC}{\partial T^2}>0$$

and
$$\frac{\partial^2 TC}{\partial t_1^2}\cdot\frac{\partial^2 TC}{\partial T^2}-\left(\frac{\partial^2 TC}{\partial t_1\partial T}\right)^2>0.$$

These values of t_1 and T will minimize the total average cost. Equations (12) and (13) can be solved with the help of software like mathematical or mat lab.

4 NUMERICAL EXAMPLES:

For the illustration of the procedure the following numerical problem is considered:

Let $\alpha=0.0001$; $\beta=2$; C_1=Rs. 10 per unit per unit time; C=Rs. 50 per unit; A=Rs.1000 per order; a=100; b=50.

The solution for optimal values of t_1 and T is given by $t_1^{\bullet}$ and $T^{\bullet}$ which gives the minimum total cost $TC^{\bullet}$.

Putting the above values in equations (12) and (13), we get

$$\frac{10}{T}\left[100\left\{2t_1-\frac{0.0004t_1^3}{3}-\frac{2.10^{-8}t_1^5}{3}+T-t_1+\frac{0.0001}{3}\left(t_1^2T-t_1^3\right)\right\}\right.$$
$$+25\left\{2t_1^2-\frac{0.0004t_1^{\beta+2}}{4}-\frac{7\times 0.0001^2t_1^{2\beta+2}}{5}\right.$$
$$\left.\left.+2t_1T-2t_1^2-0.0002\left(t_1^{\beta+1}T-t_1^{\beta+2}\right)\right\}+S\left(0.0003t_1^{\beta}-0.0002t_1T-1\right)\right].$$
$$+\frac{22500}{T}=0. \qquad \ldots(14)$$

$$-\frac{10}{T^2}\left[50\left\{2t_1^2-\frac{0.0002t_1^{\beta+1}}{3}-\frac{2\times10^{-8}t_1^{2\beta+2}}{9}\right.\right.$$
$$\left.+T^2-t_1^2+\frac{0.0001}{3}\left(-\frac{T^4}{2}-\frac{3}{2}t_1^{\beta+2}\right)\right.$$
$$+25\left\{\frac{2t_1^3}{3}-\frac{0.0001t_1^{\beta+3}}{5}-\frac{0.0001^2t_1^{2\beta+3}}{5}\right.$$
$$\left.\left.+\frac{2T^3}{3}-\frac{2}{3}t_1^3-\frac{0.0001}{2}\left(T^{\beta+3}-\frac{4}{5}t_1^{\beta+3}\right)\right\}+S\left(1-0.0001t_1^{\beta}\right)t_1\right].$$

$$-\frac{50}{T^2}\left[300t_1+75t_1^2\right]-1250=0. \qquad \ldots(15)$$

The value of S is given by

$$S=\frac{1}{\left(1+0.0001t_1{}^2\right)}\Bigg[100\left(T-t_1\right)+\frac{0.0001}{3}\left(T^3-t_1^3\right)$$
$$+50\left(\frac{T^2-t_1{}^2}{2}+0.0001\left(\frac{T^4-t_1{}^4}{4}\right)\right)\Bigg]. \qquad \ldots(16)$$

Using the value of S from (16) into the equations (14) and (15) and solving them with the help of mathematical software, we may obtain the values of $t_1^{\bullet}$ and $T^{\bullet}$. Consequently, we may obtain the optimal value of minimum total cost.

5 CONCLUSIONS:

In this paper, an order level inventory model with increasing demand and variable deterioration has presented. The demand rate has been taken as linearly increasing with time and the deterioration rate has been considered of the power pattern form. The expression for total cost has been obtained and cost minimization technique has been used to obtain the optimal values of the parameters.

REFERENCES:

1. **Aggrawal, S.P. (1978):** A Note on an Order-level Inventory Model for a System with Constant Rate of Deterioration. Opsearch, 15, 184-187.
2. **Bahari Kasani, H. (1989):** A Deterministic Inventory System with an Inventory Level Dependent Demand Rate. Jour. Oper. Res. Soc. 40, 75-81.
3. **Covert, R.P. and Philip G.C. (1973):** An EOQ Model for Items with Weibull Distribution Deterioration. AIIE Tran, 5, 323-329.
4. **Dave, U. (1986):** An Order Level Inventory Model for Deteriorating Items with Variable Instantaneous Demand and Discrete Opportunities for Replenishment. Opsearch, 23 (4), 244-249.
5. **Deb, M. and Chaudhuri, K.S. (1986):** An EOQ Model for Items with Finite Rate of Production and Variable Rate of Deterioration. Opsearch, 23, 175-181.
6. **Dave, U. & Patel, L.K. (1981):** (T, S_i) Policy Inventory Model for Deteriorating Items with Time Proportional Demand. Jour. Oper. Res. Soc. 32, 137-142.
7. **Ghare, P.M. & Scharader, G.F. (1983):** A Model for Exponentially Decaying Inventories. Jour. Ind. Eng., 14, 238-243.
8. **Hari Kishan & Mishra, P.N. (1995):** Inventory Model with Stock Dependent Demand Rate and Constant Rate of Deterioration. Acta Ciencia Indica, XXI (4M), 420-422.
9. **Hari Kishan & Mishra, P.N. (1997):** An Inventory Model with Exponential Demand and Constant Rate of Deterioration with Shortage. Ind. Jour. Math. 37, 275-279.
10. **Mandal, B.N. & Phaujdar (1989):** An Inventory Model for Deteriorating Items and Stock-dependent Consumption Rate. Jour. Oper. Res. Soc. 40 (5), 483-486.
11. **Mishra, R.B. (1975):** Optimum Production Lot Size Model for a System with Deteriorating Inventory. Int. Jour. Prod. Res. 13, 495-505.
12. **Chowdhury, Mitali Roy and Chaudhuri, K.S. (1983):** An Order Level Inventory Model for Deteriorating Items with Finite Rate of Replenishment. Opsearch.
13. **Vandana Dixit & Nita H. Shah (2006):** An Order Level Inventory Model with Decreasing Demand and Time Dependent Deterioration. Inter. Jour. Man. Sys., 22(1), 71-78.